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ABSTRACT

Exploring whether deductive reasoning can develop adequately without special instruction, this paper presents two studies that examine the development of meta-components of deductive reasoning, first in algebra, and second in verbal reasoning. The first study examined students' understanding of logical necessity in algebraic tasks in different curricular settings, where one curriculum provided instruction with an emphasis on the meta-components of algebraic reasoning and the other did not. The study involved 120 Russian and 120 English students participated in an experimental mathematics curriculum group, and 89 Russian and 120 English students participated in the nonexperimental curriculum. Each group included younger and older adolescents. Students in the experimental curriculum had a better understanding of logical necessity and this ability tended to increase with age. Students in the nonexperimental curriculum had not developed an understanding of logical necessity. In the second study, the same subjects participated in a study of the transfer of the understanding of logical necessity to verbal reasoning. The advantage noted for those in the experimental curriculum continued into the verbal reasoning tasks. (Contains 5 figures, 5 tables, and 45 references.) (SLD)

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Understanding of logical necessity in adolescents: Developmental and cross-cultural perspectives

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It has long been debated whether deductive reasoning can be taught and if so, whether the acquired reasoning skills can be successfully transferred to other contexts and content areas (Nisbett, Fong, Lehman, & Cheng, 1987). A number of proposals have been made in response to the first question. Researchers of Piagetian orientation usually maintained the view that deductive reasoning is a product of formal operational thought and, as such, it cannot be taught (Inhelder, & Piaget, 1958). However, others (e.g., Nisbett, Fong, Lehman, & Cheng, 1987; Davydov, 1962; 1990; Tulviste; 1978) attempted to demonstrate that deductive reasoning can be taught. With respect to the second question, views ranged from highly skeptical conclusions about possibilities of transfer (Thorndike, 1913), to more optimistic views (e.g., Inhelder, & Piaget, 1958; Piaget, 1953; Vygotsky, 1962; Davydov, 1990; Nisbett, Fong, Lehman, & Cheng, 1987, Girotto, Light, & Colbourne, 1988).

This paper suggests a reformulation of the question: Does deductive reasoning develop adequately without special instruction? In an attempt to answer this question, the paper examines the distinction between meta-components (e.g., the ability to distinguish between logical and empirical conclusions, understanding of logical necessity of valid conclusions, etc.) and transformational components (e.g., ability to perform necessary symbolic, linguistic, or mental transformations) of deductive reasoning. The paper (1) argues that meta-components are critical for certain types of deductive reasoning; and (2) provides evidence that meta-components do not develop naturally, but can be successfully taught in a given content area, and transferred to another content area.

One content area where deductive reasoning is critical is algebra. The paper presents two studies that examine the development of meta-components of deductive reasoning -- understanding of logically necessity, -- first, in algebra, and, second, in verbal reasoning. The first study investigates children's understanding of logical necessity in algebraic tasks in different curricular settings, where one curriculum provides instruction with an emphasis on meta-components of algebraic reasoning, while the other curricula have different curricular foci. The second study examines the transfer of the ability to understand logical necessity into the area of verbal reasoning. It is demonstrated that (1) children within the first curriculum have a better understanding of logical necessity, both in algebra and in verbal reasoning, and tend to increase this ability with age; and (2) children in the other curricula do not exhibit a natural development of understanding of logical necessity. Both groups, however, exhibited comparable performance on control reasoning tasks that did not require understanding of logical necessity.

Introduction

The study of reasoning as an integral part of human cognitive development has a long tradition in developmental psychology (e.g., see Evans, Newstead, & Byrne, 1993; Overton, 1990; Braine & Rumain, 1983, Inhelder & Piaget, 1958; Sternberg, 1982). Since Bacon, there has been a distinction between deductive and inductive reasoning. In inductive reasoning, the reasoner reaches a tentative, case-based (Moshman, 1995) conclusion based on available empirical information. Although the reasoner may follow the rules of induction, and the information in the premises may be empirically true, this does not assure an empirically correct conclusion. For example: French speak French,

Italians speak Italian, Russians speak Russian, Koreans? ... speak Korean, Americans...?. Inductive conclusions are based on the reasoner's experience, and may change when new information becomes available. If new information is consistent with the old, it increases the reasoner's belief in the conclusion, its subjective probability. However, irrespective of the reasoner's degree of certainty in the inductive conclusion, the conclusion itself is always probabilistic (e.g., see Gigerenzer & Murray, 1987, for a discussion). In contrast, in deductive reasoning, the reasoner can reach a certain conclusion based solely on information given in the premises. This conclusion, however, has to be made in accordance with the rules of deduction. Those conclusions that conform to the rules of deduction are then part of a valid deductive argument. The end-product of a valid deductive argument is a rule-based (Moshman, 1995) logically necessary conclusion that does not require additional empirical checks -- no matter how certain the reasoner is about this conclusion (see, Moshman & Timmons, 1982; Osherton & Markman, 1975 for a discussion). Thus, we need to distinguish the logical necessity of rule-based deductive conclusions and the probabilistic status of case-base inductive conclusions. Failure to make such a distinction could be a source of reasoning errors (e.g., Markman 1978.; Osherton & Markman, 1975; Fishbein & Kedem, 1982). Thus understanding of the developmental progression of the ability to understand logical necessity is critical for a better understanding of deductive reasoning.

Researchers have distinguished the transformational (e.g., linguistic, mental operational, or information-processing) level of deductive reasoning, from the meta-level (e.g., Braine, 1990; Johnson-Laird & Byrne, 1991; Moshman, 1990; Polk & Newell, 1995; Sternberg, 1982; 1984). Transformational components are necessary for an adequate deductive transformation in accordance with rules, whereas meta-components are important for the validation of a deductive conclusion. The validity of the deductive argument does not depend upon the truth value of the premises or the conclusion; thus it is possible to derive an invalid conclusion from empirically true premises, or a logically valid conclusion from empirically false premises. Logically valid conclusions do not require empirical checks. Logically valid conclusions that are based on empirically true premises, i.e., *sound conclusions*, are true *a priori*. (Those readers who wish to pursue further philosophical inquiry into this matter may be directed to Kant's *Critique of Pure Reason*, to Carnap (1966), or Quine (1953). More focused psychological discussions can be found in Evans, Newstead, & Byrne, 1993; Moshman & Timmons, 1982; Osherton, 1976; Osherton & Markman, 1975; and Overton, 1990).

It has been claimed that the ability to distinguish between valid and invalid conclusions, and between logically necessary (*a priori*) truths and empirical (*a posteriori*) truths, develops ontogenetically later than the ability to perform some deductive transformations (e.g., Braine, 1990; Osherton, 1976; Moshman, 1990). However, there is no evidence indicating <u>when</u> this ability actually develops. Those studies that specifically examined the development of understanding of logical necessity and logically necessary conclusions (e.g., Komatsu & Galotti 1986; Miller, Seier, & Nassau, 1995; Osherton & Markman, 1974; Osherton, 1976; Piaget, 1986), largely concluded that preschool and elementary school children do not understand logical necessity. Other studies (e.g., Fishbein & Kedem, 1982; Sloutsky & Morris, 1995) demonstrated that adolescents and young adults, including university students, also have difficulties in understanding logical

necessity. A number of researchers provided evidence that children, adolescents, and adults who do not understand logical necessity are still able to formulate some deductive arguments (e.g., Komatsu, & Galotti, 1986; Kuhn, 1977; Miller, Seier, & Nassau, 1995; Moshman, 1990; Osherton & Markman, 1975; Sloutsky & Morris, 1995). Some researchers have argued that the development of this ability requires exposure to literacy and schooling (e.g., Braine, 1990), whereas others maintained the view that it requires special instruction (e.g., Davydov, 1990). The ability to understand logical necessity has been attributed by various researchers to the meta-level of deductive reasoning that has been considered by researchers under different names, including meta-components (e.g., Sternberg, 1982; 1984); secondary logical skills (Braine, 1990), metalogic (Moshman, 1990; Johnson-Laird & Byrne, 1991), or metadeduction (Polk & Newell, 1995). Though different in other respects, Braine's (1990), Moshman's (1990) and Johnson- Laird & Byrne's (1991) models of deduction all suggest that the meta-level is an important part of deductive reasoning, specifically of the validation of deductive conclusions. Throughout this paper we will use the term meta-components in order to refer to the meta-level of deduction.

Understanding of logical necessity is important for reasoning in many domains, including verbal reasoning, scientific reasoning, and mathematical reasoning, as well as in some other areas of verbal discourse (e.g., communication). Failure to understand logical necessity, and subsequent confusion of logically necessary and probabilistic conclusions, could lead to serious reasoning errors in children and adults, across various reasoning domains. These errors, among others, may include belief biases in verbal reasoning (e.g., Evans, Newstead, & Byrne, 1993), confusion of proof and evidence in mathematical reasoning (e.g., Fishbein & Kedem, 1982; Morris, 1995), and disrationalia (e.g., Stanovich, 1993).

One content area where understanding of logical necessity is critical is mathematics, specifically, algebra and geometry. Understanding of the givens as premises, proof as the deductive argument, and the theorem as the logically necessary conclusion from the proof largely defines mathematical reasoning. There is a significant body of evidence demonstrating the importance of understanding of logical necessity in algebraic reasoning (e.g., Davydov, 1990; Fishbein & Kedem, 1982; Morris, 1995; Vinner, 1983). Thus, algebraic reasoning was considered to be an appropriate content area in which to examine the posed research questions. In addition, algebra is taught in school to most children; therefore simple algebraic reasoning problems are familiar to most children. Finally, algebra is taught in accordance with a certain rationale. Some algebra curricula are based on an eclectic combination of principles, whereas others have explicit priorities with respect to the development of various components of algebraic deduction (Morris, 1995). Thus the comparison of the algebraic reasoning of children who were specifically taught to understand meta-components, with those who were primarily taught to perform transformations, could provide answers to the research questions. Do children develop understanding of logical necessity without a special instruction? Does special instruction in algebra facilitate the development of understanding of logical necessity, so that children who do and do not receive special instruction in understanding meta-components, demonstrate different performances on algebraic tasks that require understanding of

logical necessity? And do all, or some of these differences carry over to another content area, namely, verbal reasoning?

Theoretical positions proposed by this paper can be summarized as follows. First, understanding of logical necessity, as ability to distinguish logically necessary *a priori* conclusions from empirical *a posteriori* conclusions, is considered to be an important component of the reasoning process. Second, many children, adolescents, and adults do not naturally develop understanding of logical necessity. Third, instructional emphasis on meta-components can amplify reasoning on tasks that require understanding of logical necessity within a specific content area; i.e., it is possible to teach understanding of logical necessity. And fourth, understanding of logical necessity learned within one content area can be transferred to another content area.

These positions led us to formulate the following hypotheses:

Understanding of logical necessity does not develop naturally; it requires special instruction. Therefore,

1. Children who receive instruction with specific emphasis on meta-components of deductive reasoning will exhibit a greater understanding of logical necessity within algebra, than children who do not receive such training.
2. Children who receive instruction in understanding of meta-components within algebra, will outperform others on algebraic reasoning tasks that require understanding of logical necessity.
3. Children who do not receive instruction in understanding of meta-components will exhibit a slower developmental progression in understanding of logical necessity within algebra than children who receive such instruction.

Transfer of understanding of logical necessity from one content area to another is possible. Therefore,

4. Children who receive instruction in understanding of meta-components within algebra will outperform others on verbal reasoning tasks that require understanding of logical necessity.

In order to test these hypotheses, two studies were conducted. The first study examines the development of understanding of logical necessity within algebra. The second study examines the transfer of this ability into the area of verbal reasoning.

Both studies included children participating in different algebra curricula. The first curriculum is an "experimental" elementary mathematics curriculum in Russia (Davydov, 1962; 1975; 1990). This curriculum emphasizes teaching of principles of abstract deductive reasoning prior to concrete numerical work. It has a specific emphasis on meta-components, and the distinction of logically necessary (*a priori*) conclusions from empirical (*a posteriori*) conclusions (Davydov, 1990). It also emphasizes the logical necessity of particular (e.g., numerical) cases that are derived from general mathematical principles and relationships, where those principles and relationships are first expressed algebraically. Concepts of quantity, relations, and structure are developed and emphasized prior to numerical work, and prior to emphasis on algebraic transformations. As this is an elementary school curriculum, it lays the groundwork for the acquisition of

transformational components during a middle school algebra course. The second "experimental" curriculum has a diametrically opposed theoretical orientation. National Mathematics Project (NMP) in England (Harper, Kuchemann, et al., 1987) tends to replicate a natural progression in development of algebraic reasoning -- the progression from more concrete transformational components to more abstract meta-components, from computation to understanding of abstract algebraic principles. Transformational components are taught in an inductive, case-based manner, via the investigation of a number of particular instances. The validity of algebraic generalizations is assessed via empirical checks. (For a more comprehensive discussion of both curricula, see Morris, 1995.)

As these curricula were implemented in different cultures, curricular variables were confounded with other potentially important variables such as language, family processes, and cultural beliefs and practices -- variables that may affect academic performance (Chapman, Skinner, & Baltes, 1990; Hess, Azuma, Kashiwagi, Dickson, Holloway, Miyake, Price, Hatano, & McDevitt, 1986; Okagaki, & Sternberg, 1993; see also Murphey, 1992, for a review). To control potential cross-cultural confounds, and to allow within-cultural as well as cross- cultural comparisons, schools using "non-experimental" curricula were selected in the same countries. These schools were in the same geographical area, and had comparable student and teacher populations; however, they did not have curricula that were designed to develop specific kinds of algebraic reasoning.

The non-experimental curriculum in Russia has more emphasis on deduction than both English curricula; however, it starts developing deductive reasoning later than the Russian experimental curriculum (Davydov's curriculum) and it does not have a specific emphasis on understanding meta- components, such as underlying logical principles. The algebra course for grades 7-9 is "characterized by the enhancement of the theoretical level of instruction and by the stronger emphasis that is gradually placed upon the role of theoretical generalizations and deductions" (USSR Academy of Pedagogical Sciences' Scientific Research Institute of Curriculum and Teaching Methods, 1987, p. 67). The English non-experimental curricula has been characterized as inductive; there is a general emphasis on deriving generalizations from an examination of particular cases (see, e.g., Bell, 1976). This curriculum tends to reduce the emphasis on formal algebra by using more numerical problem solving.

In order to provide a better understanding of cross-sectional differences in understanding of logical necessity-differences that could point to specific developmental mechanisms-children of two age groups were included in the design: (1) early adolescents, and (2) middle and late adolescents. Early adolescents (younger group) had at least one year of exposure to formal algebra study, whereas middle and late adolescents (older group) had experienced at least three years of formal algebra studies.

As the curricula were implemented under very different conditions in different educational systems, the reported findings do not allow us to judge the quality, success, or educational virtues of the curricula; or to make specific curricular recommendations. Rather the results allow us to provide some preliminary answers to the posed research questions.

Study 1. Understanding of logical necessity within the domain of algebra

Method

Subjects

For purposes of comparison, four groups were included in the sample: (1) Russian experimental -- students and graduates of Davydov's experimental elementary school mathematics curriculum, implemented in Moscow School #91 in Moscow, Russia (n=120); (2) Russian non-experimental -- students in a non-experimental school in Moscow, Russia (n=89); (3) English experimental -- students in an upper school in England that had implemented the National Mathematics Project curriculum (n=120); and (4) English non-experimental -- students in an upper school in England with a "non-experimental" curriculum (n=120). Each group included children at two age levels -- early adolescents ("younger group") and middle and late adolescents ("older group"). Details of the sample are described in Table 1.

Table 1. Detailed description of the sample.

	ENGLAND	RUSSIA
Experimental curricula		
Younger group.....	N=32; MEAN=13.719; S.D.=0.457	N=62; MEAN=11.887; S.D.=0.770
Older group.....	N=88; MEAN=15.216; S.D.=0.718	N=59; MEAN=15.288; S.D.=0.645
Non-Experimental curricula		
Youngergroup.....	N=35; MEAN=13.771; S.D.=0.426	N=55; MEAN=12.109; S.D.=0.599
Older group.....	N=85; MEAN=15.247; S.D.=0.688	N=34; MEAN=14.941; S.D.=0.600

The selection of these age groups was determined by two factors. The first consideration was that children in the younger group, under each condition, would have at least one year of exposure to formal algebra; and that children in the older group would have at least three years of formal algebra instruction. This attempt to match children's exposure to algebra resulted in the following: for the younger group, children in the Russian samples are a year younger than their English peers (see Table 1). This is due to the fact that in Russia, algebra begins a year earlier than in England. This was not considered to be a problem for the study, however, because the direction of performance differences was hypothesized to be opposite to the direction of age differences.

Sample selection was determined by the implementation of both experimental curricula. First, schools that implemented the experimental curricula were selected in both countries. Then, non-experimental schools were selected in a similar geographic location. The criterion for selection was that schools would have comparable student and teacher populations. The schools were selected in middle-class neighborhoods. Each school was contacted by a member of a research group. After permission to conduct the study was granted, the researchers contacted children of the respective age levels. All contacted children agreed to participate in the study. The samples were randomly selected from lists of students in each respective school. Sample selection in Russia was arranged and overseen by the authors through the International Laboratory for Comparative Social Research, established by Ohio State University and the Russian Academy of Education. Sample selection in England was carried out by one of the authors.

Instruments

Instruments were designed to compare children's performance on tasks that required understanding of logical necessity. In order to control the overall level of algebra proficiency, algebraic tasks that do not require understanding of logical necessity were also included in the design. All instruments were prepared in English and then translated into Russian. After a back translation, necessary revisions were made in the Russian equivalents of the tasks. The following tasks were used in this study.

Tasks that required understanding of logical necessity.

(1) VICTOR PROBLEM (Fishbein & Kedem, 1982).

> In algebra class the teacher proved that every whole number of the form n^3-n is divisible by 6 with no remainder. The proof was as follows:
> We can write:
> $n^3-n = n(n^2-1)$
> Then we can rewrite the expression on the right:
> $n(n^2-1)=n(n-1)(n+1)$
> So:
> $n^3-n = n(n-1)(n+1) = (n-1)(n)(n+1)$
> But (n-1)(n)(n+1) is a product of three consecutive whole numbers. Therefore one of them should be divisible by 2, and one of them (not necessarily a different one) should be divisible by 3. Thus their product should be divisible by 2 x 3, that is, by 6.
>
> However, Victor (Petya, in the Russian version) is a doubter. He thinks that we have to check at least a hundred numbers in order to be sure that the theorem is correct. What is your opinion? Explain your answer.

This task specifically measured understanding of proof as a logically necessary argument. The task has been used in previous research (e.g., Fishbein & Kedem, 1982; Vinner, 1983).

(2) GIRL PROBLEM (Lee & Wheeler, 1989).

> A girl multiplies a number by 5 and then adds 12. She then subtracts the original number and divides the result by 4. She notices that the answer she gets is 3 more than the number she started with. She says, "I think that would happen, whatever number I started with." Is she right. Explain, why your answer is right.

This task specifically measured children's ability to apply their understanding of logical necessity, and to formulate a formal proof as a deductive conclusion. This task has been used in previous research (e.g., Lee & Wheeler, 1989).

Control algebraic tasks.

(1) STUDENT-PROFESSOR PROBLEM (Clement, Lockhead, & Monk, 1981).

> Write an equation using the letters S and T to represent the following statement: "It here are six times as many students as teachers in this school." Use S for the number of students and T for the number of teachers.

This task requires a certain level of algebraic proficiency, but does not require understanding of logical necessity. The student-professor problem has been used extensively in previous research; it has been demonstrated that even college students experience difficulty in writing correct equations in response to this task (see, e.g., Bernardo & Okagaki, 1994; Clement, Lochhead, & Monk, 1981; Fisher, 1988; Hegarty, Mayer, & Green, 1992; Lewis & Mayer, 1987; Mestre, 1988; Rosnick & Clement, 1980; Schoenfeld, 1985; Spanos, Rhodes, Dale, & Crandall, 1988).

Procedure

The tasks were presented as a written test that was distributed by a research assistant or specially trained the regular classroom teacher, during the regular math class. The tasks were part of a larger test. Children were given 90 minutes for the entire test. After the test, children were interviewed. Children were asked to explain their solutions to each problem. Interviews took place in a separate room in the school.

Test and interview data were encoded in accordance with a coding catalog. Two coders encoded children's responses to the problems. Cohen's kappas, indices of inter-rater reliability, varied for the described tasks from .96 to 1.

Results and discussion

The data were cross-tabulated (culture-curricular composite * age * response category), and subjected to log-linear analyses. "Culture-curricular composite" represents the four groups: (1) Russian experimental curriculum; (2) English experimental curriculum; (3) Russian non-experimental curriculum; and (4) English non-experimental curriculum. The analyses allowed us to select appropriate well fitting log-linear models; and to determine goodness-of-fit chi-squares, component chi-squares, p-values of the models and of the effects, directions of the effects, z-scores for the effects (λ/s.e.), and effect sizes (ϕ).

In response to the Victor problem, children provided three main categories of responses, including: (1) I agree with Victor, empirical checks can not prove anything; or because the algebraic proof is sufficient. (2) I disagree with Victor because empirical checks may be too many. (3) Other responses. Percentages of responses falling into each response category, and examples of the responses within each category, are provided in table 2.

Table 2. Percentages of responses in each response category and examples of responses in the Victor problem.

	Empirical checks are needed	Proof is sufficient	Other response
Younger group			
Russian experimental.....	8%	4%	88%
Russian non-experimental..	13%	0%	87%
English experimental.....	67%	6%	27%
English non-experimental..	65%	0%	35%
Older group			
Russian experimental.....	20%	60%	20%
Russian non-experimental..	18%	11%	71%
English experimental.....	73%	12%	16%
English non-experimental...	55%	11%	34%
Examples of answers falling into each category...	I think that Victor is right to be doubtful. Just because it works for some numbers does not mean that it works for the others.	Bad method of checking the theorem, because it might be correct for 100 numbers and incorrect for the 101st. Petya is fool because the theorem is proven analytically.	(1) I did not understand the proof. (2)Petya has an inferiority complex -- he needs to see a psychotherapist.

Agreement with Victor suggested that the child confused proof as logical necessity with an empirical inductive statement; whereas disagreement with Victor accompanied by an adequate explanation was interpreted as understanding of proof as logical necessity. Figure 1 compares percentages of children who responded that it is necessary to check the proof empirically, versus those who suggested that proof is sufficient.

Insert Figure 1 around here

Data in the table and in the figure demonstrate that older adolescents in the Russian experimental group exhibited a better understanding of proof as logical necessity than children in any other group, whereas the majority of children in the English groups suggested that the proof requires additional empirical checks. For the first response category (empirical checks are needed), the model with effects due to culture-curricular composite fit the data well (χ^2 (4)=3.89, p=.42; component χ^2 (3)=133.1; p<.0001; ϕ (effect size) = .55). The direction of the effects suggested that children in the English groups were more likely to request empirical checks than children in the Russian groups (z=8.3; p<.0001). Children in the English experimental curriculum were more likely to request empirical checks than children in any other group (z =2.5; p<.01). Data also suggests an absence of developmental effects: in all four groups, children neither increased, nor decreased their tendencies to request empirical checks for the proved theorem.

For the second response category (proof is sufficient), the model with effects due to culture-curricular composite and age group fit the data well (χ^2 (3)=4.63, p=.2) Effects due to group were moderate to large (component χ^2(3)=50.1, P<.0001; ϕ (effect size) =

.33). Effects due to age were large (component $\chi^2(2)=324.4$, $P<.0001$; ϕ (effect size) = .87). The direction of the effects suggested that older children in the Russian experimental curriculum were more likely to treat proof as logical necessity than children in any other group ($z=4.5$; $p<.0001$). Children in the Russian experimental group were also more likely to increase their understanding of logical necessity with age ($z=10.6$; $p<.0001$), whereas children in other groups did not exhibit such an increase.

In response to the girl problem, children provided two main categories of responses, including: (1) formulation of an algebraic proof; and (2) other response (e.g., no response, use of numerical examples only). Percentages of responses falling into each response category and examples of responses within each category are provided in Table 3.

Table 3. Percentages of responses in each response category in the girl problem.

	Used algebraic proof	Other response
Younger group		
Russian experimental.....	19%	81%
Russian non-experimental.	11%	89%
English experimental.....	1%	99%
English non-experimental..	1%	99%
Older group		
Russian experimental.....	69%	31%
Russian non-experimental.	41%	59%
English experimental.....	9%	91%
English non-experimental..	2%	98%

Figure 2 compares percentages of children who used an algebraic proof in response to this task, versus those who did not.

Insert Figure 2 around here

Children's solutions to the girl problem demonstrates that older children in the Russian experimental group exhibited a better understanding of algebraic proof. For use of proof as a response category, the model with effects due to culture-curricular composite and age fit the data well (χ^2 (3)=1.05, p=.71). Effects due to the cultural-curricula composite were large (component χ^2 (3)=115.2; $p<.0001$; ϕ (effect size) = .51). Effects due to age were moderate to large (component χ^2 (3)=86; $p<.0001$; ϕ (effect size) = .44). The direction of the effects suggested that older children in the Russian experimental group were more likely than children in any other group to use algebraic proof as a solution to the girl problem ($z=4.5$; $p<.0001$); and children in the Russian non-experimental group were more likely to use proof than children in English groups ($z=6.8$; $p<.00001$). Children in the Russian groups were more likely to increase their use of proof over age levels than children in other groups ($z=12.4$, $p<.000001$). Children in the Russian experimental group dramatically increased their use of algebraic proof (from 19% to 69%), whereas children in the English groups only minimally (from 1% to 9%) increased their use of algebraic proof. However, the student-professor problem, -- the control task that did not require understanding of logical necessity -- yielded more comparable results. Table 4 presents percentages of children who wrote a correct

equation in response to the student-professor problem, versus those who provided other responses.

Table 4. Percentage of children writing correct equations versus other responses in the student-professor problem

	Younger group	Older Group
Russian Experimental.....	60%	83%
English Experimental.....	63%	60%
Russian Non-Experimental.....	20%	53%
English Non-Experimental.....	11%	28%

Figure 3 presents the developmental progression in children's performance on the student-professor task.

Insert Figure 3 around here

First, more than 50% of children in each experimental curriculum performed successfully on this task (95% confidence interval are 51% to 74%, for the English group; and 59% to 80%, for the Russian group). Differences were largely attributable to the non-experimental groups. The fully saturated model fit the data. Effects due to age were moderate (component χ^2 (3)=77; $p<.0001$; ϕ (effect size) = .41); effects due to the composite were also moderate (component χ^2 (3)=70; $p<.0001$; ϕ (effect size) = .39); and effects due to interaction were small (χ^2 (3)=8; $p<.001$; ϕ (effect size) = .13). Even though effects due to group were sizable, they largely explained the differences between the Russian and English non-experimental groups. The interaction term suggests that for the experimental groups, effects differed across age levels. In the younger group, there was no difference between the Russian and English experimental groups, whereas for the older group, children in the Russian experimental group were more likely to write a correct equation ($z=3.4$; $p<.001$).

The findings of this study suggest that within algebra, older children in the Russian experimental group are more likely to develop an understanding of logical necessity, to interpret proof as logical necessity, and to apply this understanding in algebraic reasoning tasks. English children are more likely to confuse logical necessity with empirical fact, and to request empirical support for logically necessary conclusions. However, both groups exhibit relatively high performance on the control task, with more than 50% of children in the English and Russian experimental groups performing successfully on the control task. It is also important to note that children in the Russian experimental group exhibited a developmental progression, significantly increasing their performance on all the tasks over age, whereas children in the English experimental group did not exhibit such a progression.

In order to test children's transfer of understanding of logical necessity to the content area of verbal reasoning, a second study was carried out.

Study 2. Children's understanding of logical necessity within the domain of verbal reasoning.

Method

Subjects

The same groups of subjects that participated in the first study also participated in the second study.

Instruments

As in the first study, instruments for this study compared children's performance on tasks that did and did not require understanding of logical necessity. The instruments were prepared in English and then translated into Russian. After a back translation, necessary revisions were made in the Russian equivalents of the tasks. In this study, the instruments evaluated children's performance on verbal reasoning tasks.

For the study of understanding of logical necessity, a special verbal reasoning task was developed. The task was presented as a syllogism, where the logically necessary conclusion was empirically wrong. Tasks that did not require understanding of logical necessity included different forms of verbal syllogisms.

Tasks that required understanding of logical necessity.

The following task suggests a conflict between a logically valid and an empirically true conclusion.

(1) FAHMOOTH NUMBER.

> Assume that the first two sentences are true. Make a conclusion from the assumptions (Choose a, b, c, or d).
>
> All fahmooth numbers can be divided evenly by 8.
> 26 is a fahmooth number
>
> Therefore:
> a) 26 must not be a fahmooth number
> b) 26 is an exception to the rule
> c) It is probably true that fahmooth numbers cannot be divided evenly by 8.
> d) 26 can be divided evenly by 8.

Control tasks. The tasks were adapted from Wilson, Cahen, & Begle (1966). Children were presented with three syllogisms. These syllogisms differed with respect to the form of their premises (positive vs. negative, and universal vs. particular) and the validity of their conclusion (valid vs. invalid). These tasks had the following instructions:

> Please tell whether or not the sentences show correct reasoning. All the sentences are really nonsense, but you are to think only about the reasoning. Circle YES if the reasoning is good, and NO if the reasoning is not good.

(1) POSITIVE-UNIVERSAL (valid).

> If all birds have purple tails and all cats are birds, then all cats have purple tails.

(2) POSITIVE-PARTICULAR (valid)

> If some men are purple and everything which is purple is a horse, then some horses are men.

(3) NEGATIVE-UNIVERSAL (invalid)

> If no skunks have green toes and all skunks are pigs, then no pig has green toes.

Procedures

The same as in the previous study.

Results and discussion

These data were cross-tabulated (culture-curricular composite * age * response category) and subjected to log-linear analyses. The analysis allowed us to select appropriate well fitting log-linear models, to determine goodness-of-fit chi-squares, component chi-squares, p-values of the models and of the effects, directions of the effects, z-scores for the effects (λ/s.e.), and effect sizes (ϕ). Percentages of children correctly responding to the verbal reasoning problems are presented in table 5.

Table 5. Percent of children correctly responding to the verbal reasoning problems.

	Fahmooth	Positive-Universal	Positive-Particular	Negative-Universal
Younger group				
Russian experimental.....	3%	71%	35%	32%
Russian non-experimental....	9%	44%	56%	24%
English experimental.....	19%	78%	43%	34%
English non-experimental....	14%	91%	37%	20%
Older group				
Russian experimental.....	30%	86%	51%	47%
Russian non-experimental....	12%	74%	32%	9%
English experimental.....	19%	91%	35%	35%
English non-experimental....	9%	86%	44%	27%

The data suggests that children perform differently on different types of syllogisms: their performance on the positive-universal valid syllogisms was higher than on syllogisms of other types (see Figure 4). Children in the Russian and English experimental groups did not differ significantly with respect to their performance on the control tasks.

Insert Figure 4 around here

For all groups, children did not increase their performance on the control tasks with age. That was not true for the fahmooth problem. Children in the Russian experimental group significantly increased their performance with age (see Figure 5). While no significant differences were found for the younger groups, older children in the Russian experimental group performed significantly better than children in other groups ($z=3.3$; $p<.0001$),

significantly increasing their performance over age levels ($z=4.6$; $p<.00001$). At the same time, children in other groups did not exhibit any developmental progression.

Insert Figure 5 around here

The results of the second study indicate that in the area of verbal reasoning (that is not a school subject), older children in the Russian experimental group exhibited better performance on tasks that required understanding of logical necessity than children in any other group. At the same time, no significant differences were found between the Russian experimental and the English experimental groups on verbal reasoning tasks that did not require understanding of logical necessity.

General Discussion

The presented data fit the hypotheses well. As was predicted by the first, second, and third hypotheses, children who received special instruction in understanding of meta-components of algebraic deductive reasoning, did in fact outperform other children on tasks that required understanding of logical necessity. In the Victor problem, 60% of the older children in the Russian experimental group suggested that proof is sufficient, whereas similar responses of older children in other groups were in the 11%-12% range. At the same time 55%-73% of the English groups suggested that empirical checks are needed, and 71% of older children in the Russian non-experimental group did not answer this question. Children who did not receive instruction in understanding of meta-components of algebraic deductive reasoning did not increase their understanding of logical necessity with age, whereas those who did receive such instruction dramatically increased their understanding of logical necessity with age.

There were also significant differences among children with respect to application of their understanding of logical necessity. On the girl problem, children in the Russian experimental group performed significantly better than children in any other group; and exhibited the largest increases in performance across age levels. At the same time, children in other groups did not exhibit such a developmental progression. Thus the data suggest that for the Russian experimental group, understanding of logical necessity tended to increase with age; whereas for children in other groups, this was not the case. Since children in the Russian experimental group outperformed children in all other groups, including the Russian non-experimental group, this allows us to eliminate potential cross-cultural interpretation of findings. At the same time, the control task yielded much more comparable results between children in the Russian and English experimental groups.

The fourth hypothesis predicted that children can transfer their understanding of logical necessity from algebra to verbal reasoning. In fact, findings indicated that for the task that required understanding of logical necessity, older children in the Russian experimental group performed better than children in any other group. Furthermore, performance significantly increased with age only for the Russian experimental group. At the same time children in other groups did not increase their understanding of logical

necessity in verbal reasoning with age. No such differences were found with respect to the control verbal reasoning tasks.

It should be pointed out that the four hypotheses were based on the implicit assumption that differences are in fact attributable to instructional variables. The findings provide a strong argument for this assumption. First, there was no indication that results were due to purely cross-cultural differences (English groups vs. Russian groups). Second, differences between the Russian experimental group and other groups tended to increase with children's age for tasks that require understanding of logical necessity; and to not increase for tasks that do not require such an understanding. However, additional studies with better control of children's intelligence levels are needed, in order to test potential rival explanations.

The results allow us to reach the following tentative conclusions:

1. Children do not exhibit natural development of understanding of logical necessity.
2. Children who received special instruction in understanding of meta-components of algebraic deductive reasoning, exhibit better understanding of logical necessity, and better application of this understanding in algebraic reasoning tasks, than children who did not receive such instruction.
3. Differences in understanding of logical necessity between children who received special instruction in understanding of meta-components in algebraic reasoning, and other children, tended to increase with children's age.
4. Understanding of logical necessity can be transferred from algebraic to verbal reasoning.

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Figure 1. Solutions to the Victor problem: Proofs vs. empirical checks

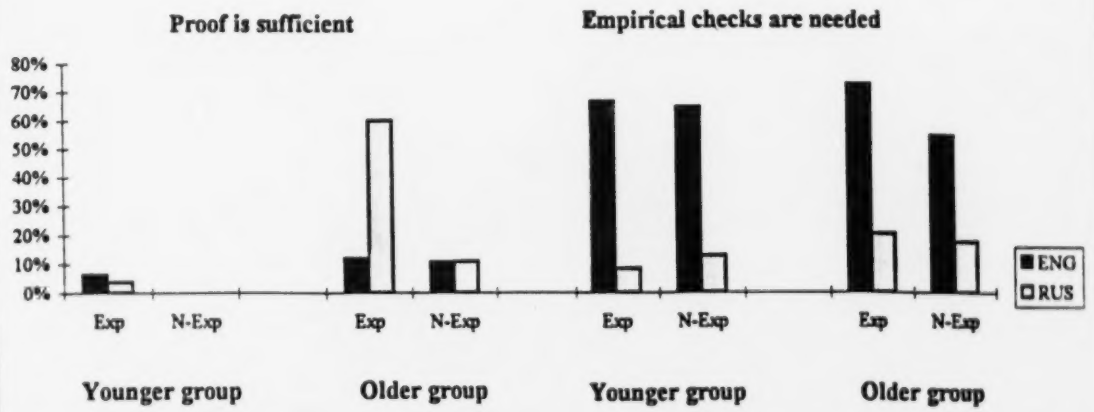

Figure 2. Use of proof in the girl problem

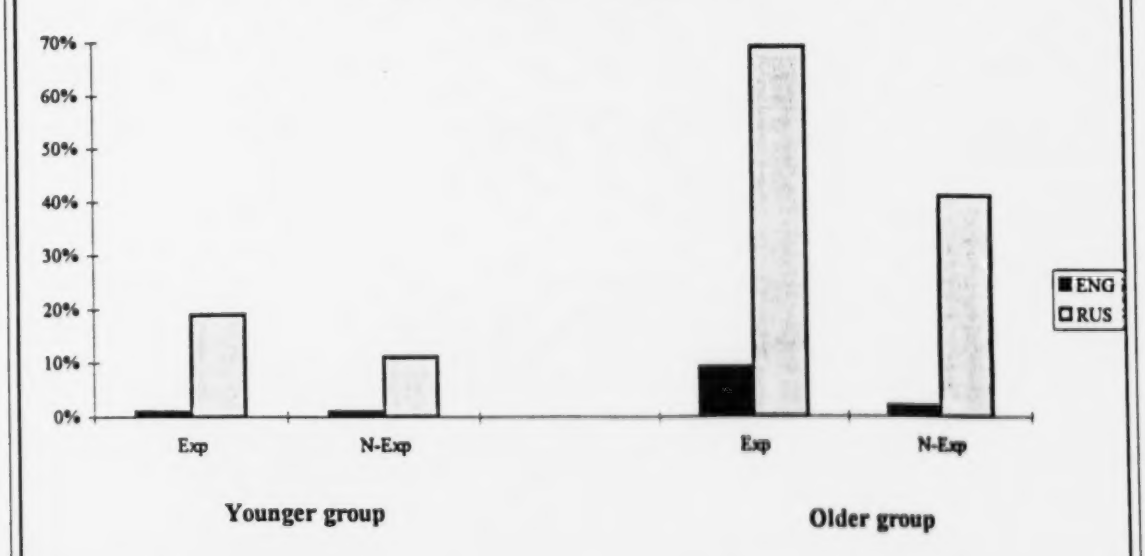

Figure 3. Children's performance on student-professor problem

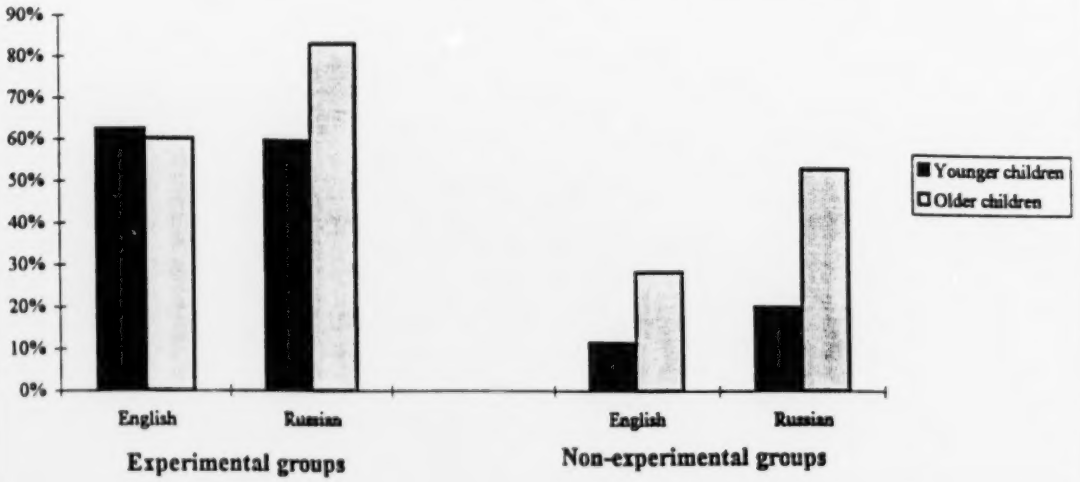

Figure 4. Verbal reasoning by age level and type of syllogism

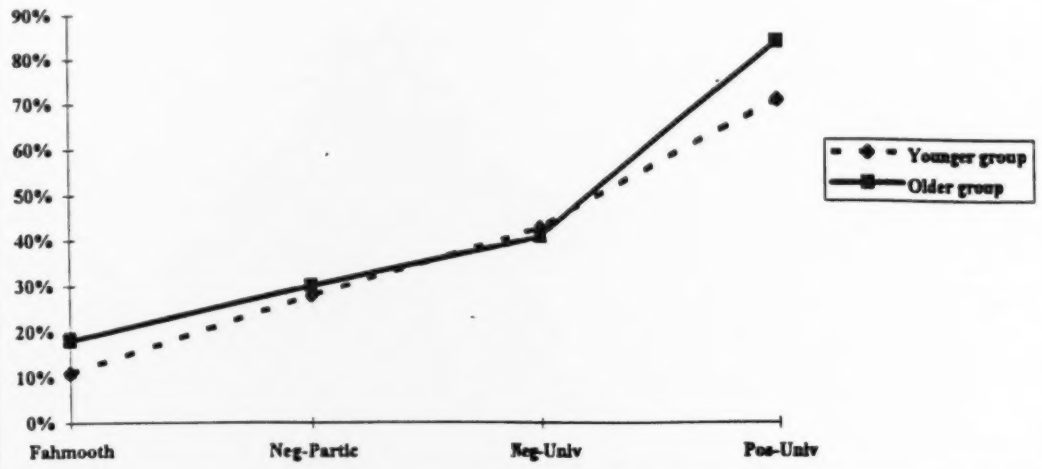

Figure 5. Correct solutions to the fahmooth problem